Engs 26: Control Theory
Spring 2017

Duck Car Project Report

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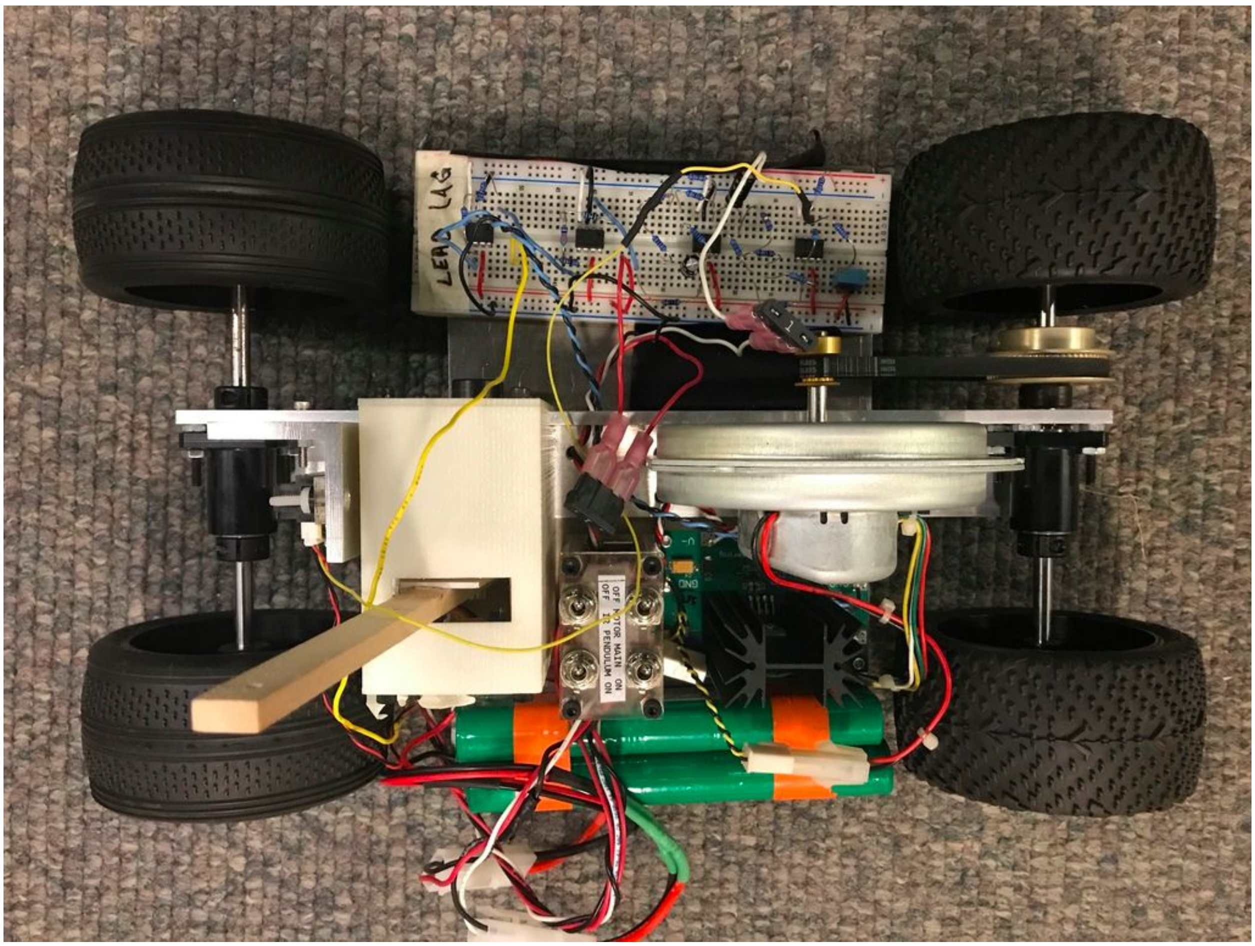

I. Introduction

The primary goal of this project was to design a compensator for a car equipped with a DC motor, IR sensor and a maximum output of 12V on the power amp. The compensated system must maintain a fixed distance from an object placed in front of the car regardless of whether the object is moving or stationary. The compensated system must also fulfill the specifications below.

- Error steady state must be zero
- System must be stable with a minimum 6 dB gain margin and 30 degrees phase margin
- Transient response to a step input must be fast and well-damped
- Max magnitude for closed loop frequency response is 1 Db above DC value
- Closed-loop bandwidth must be increased
- System reduces response to disturbances and uncertainty

Given these requirements, we settled on the design specifications of **settling time** less than or equal to **3 seconds** and **percent overshoot** less than or equal to **25%**.

II. Initial System Characterization

Overall Closed-Loop System Layout

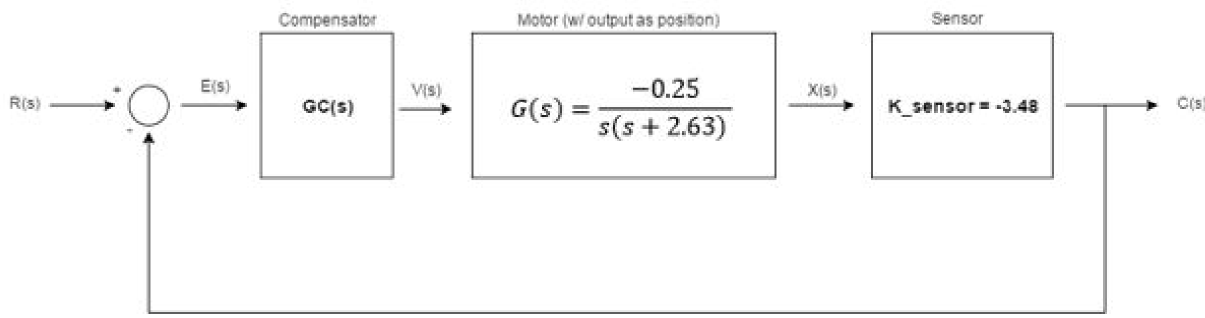

Figure 1: Block diagram of the closed loop system with unspecified compensator

Before analyzing the specifics of our system, we developed an overall block diagram for the closed loop system (see Figure 1 above). Our closed loop system consists of a compensator which outputs a voltage V(s) to drive the duck car's motor. The motor changes the position of the car X(s) and the sensor then measures how far the car is from any objects and outputs a voltage proportional to that distance. The voltage is then sent through a summing junction and subtracted from a reference voltage to speed up or slow down the car based on how far it is from any obstacle and ensure it stops a set distance from the object. See Table 1 below for more details on signals in the diagram above. In the following sections, we explain how we determined the transfer functions in each of the blocks above.

Table 1: Signal Explanations

Signals	Explanations
R(s)	Reference voltage signal - the voltage output of the sensor when the car is the distance from an object at which it is supposed to stop.
E(s)	Error signal - output of the summing junction; determines the control effort of the compensator to correct the error in the closed-loop system
V(s)	Voltage output of the compensator which drives the motor
X(s)	Change in position of the duck car that results from the motor moving the car
C(s)	Voltage output of the sensor corresponding to how far the duck car is from an object

Obtaining Sensor Gain

In order to characterize the IR sensor, we started with the car's IR sensor 4 inches away from the object and measured the voltage output of the sensor, we took voltage measurements at various distances between 4 inches and 23 inches to obtain a curve as shown:

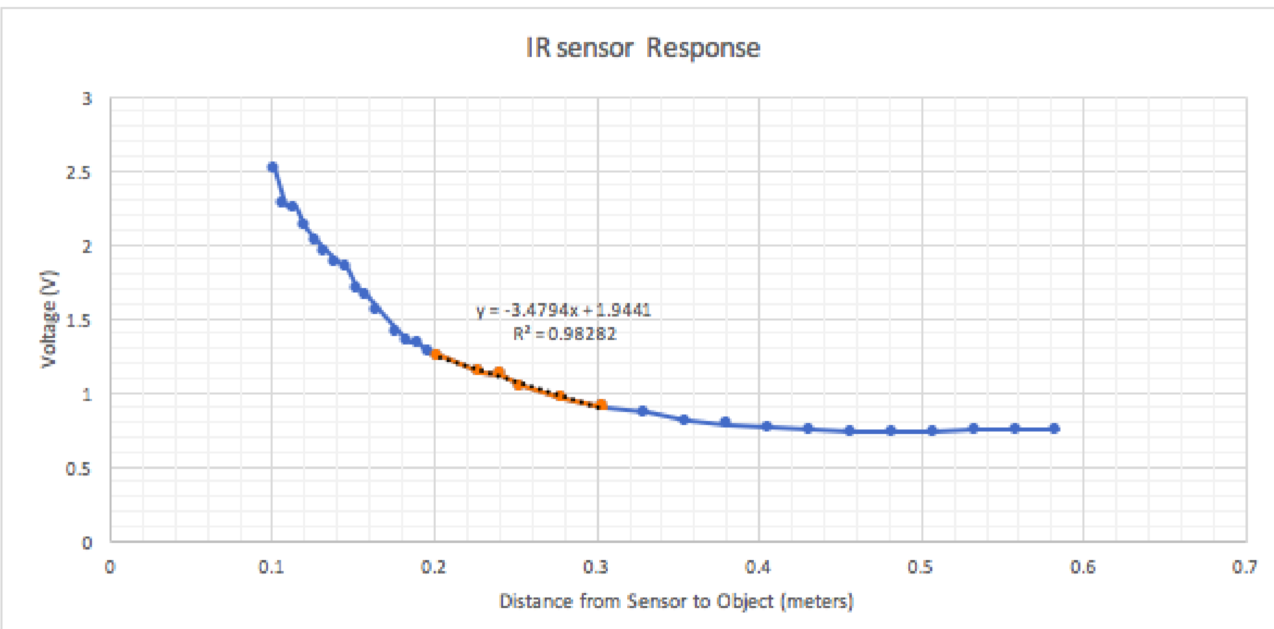

Figure 2: IR Sensor Characterization

We arbitrarily decided to linearize the curve about 10 inches or 0.2 meters by obtaining a line of best fit among the values from 8 inches (0.2 meters) to 12 inches (0.3 meters). We chose this region since it appeared to be quite linear. The slope of the line of best fit was **-3.48 volts/meter**. This is the value of our system's sensor gain required for converting change in position to a voltage value.

Obtaining Motor Transfer Function

In order to determine the the transfer function for the DC motor, we used an oscilloscope to measure how changes in voltage and frequency from the function generator affects the output of the tachometer. We applied a modular square wave as a step input to ensure that the car would pause before changing direction, this allowed us to obtain a more accurate response for the DC motor. For different inputs, we measured the voltage at which steady state was reached for the first order DC motor system and found time constant to use in the following transfer function for a DC motor:

$$G(s) = \frac{k}{(s + \frac{1}{T})}$$

A few of our most promising measurements using different inputs from the function generator are shown below. All of these outputs measure the forward response with a step input, and Tau represents the time constant measured at 63% of the steady state voltage.

Measurement #1

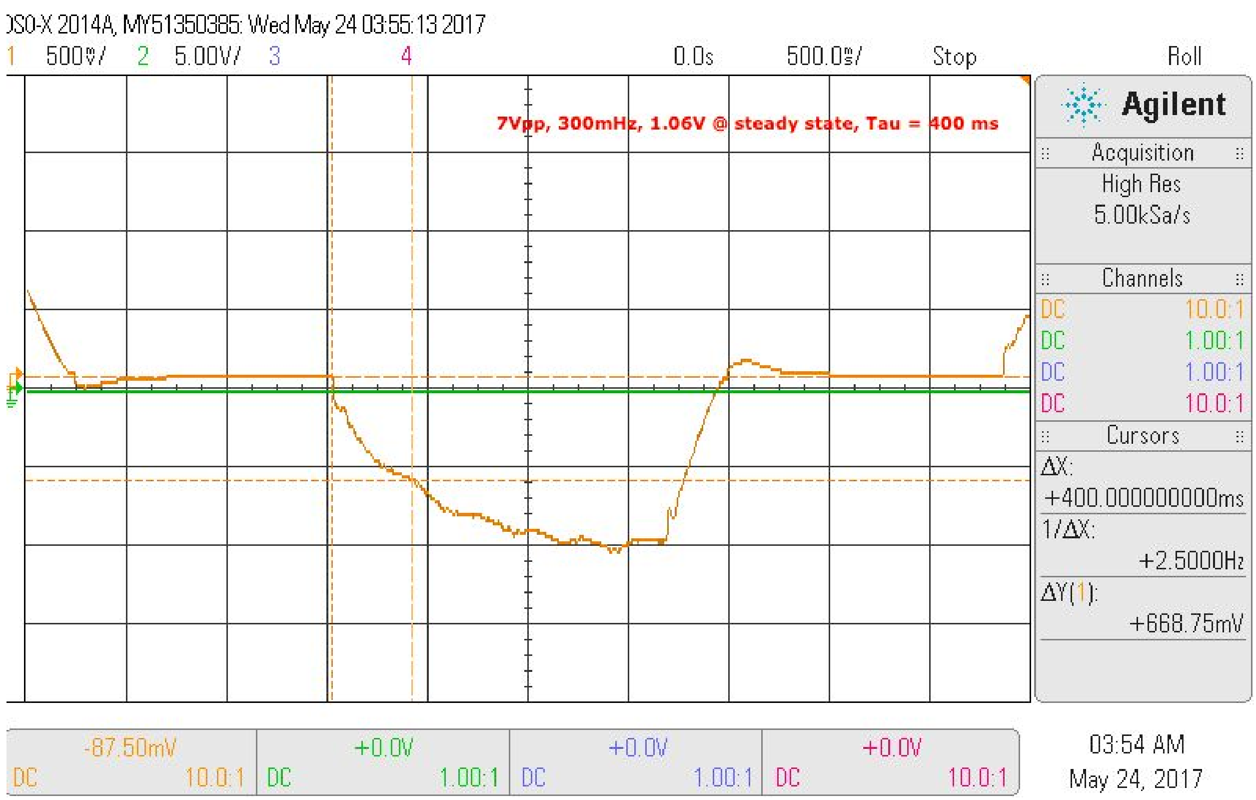

Figure 3: Oscilloscope measurement of tachometer output voltage corresponding to motor velocity in response to a -3.5 V step input. The steady state output is 1.05 V and the time constant is around 400 ms.

Measurement #2

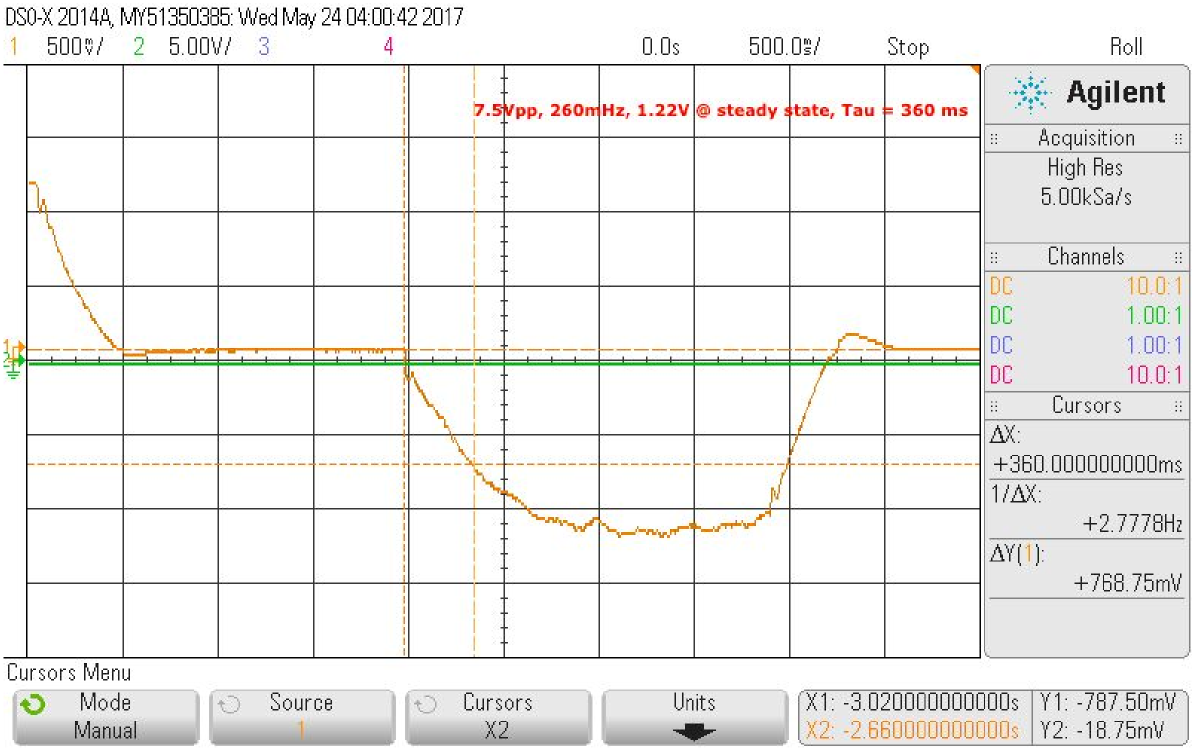

Figure 4: Oscilloscope measurement of tachometer output voltage corresponding to motor velocity in response to a -3.75 V step input. The steady state output is 1.22 V and the time constant is around 360 ms.

Measurement #3

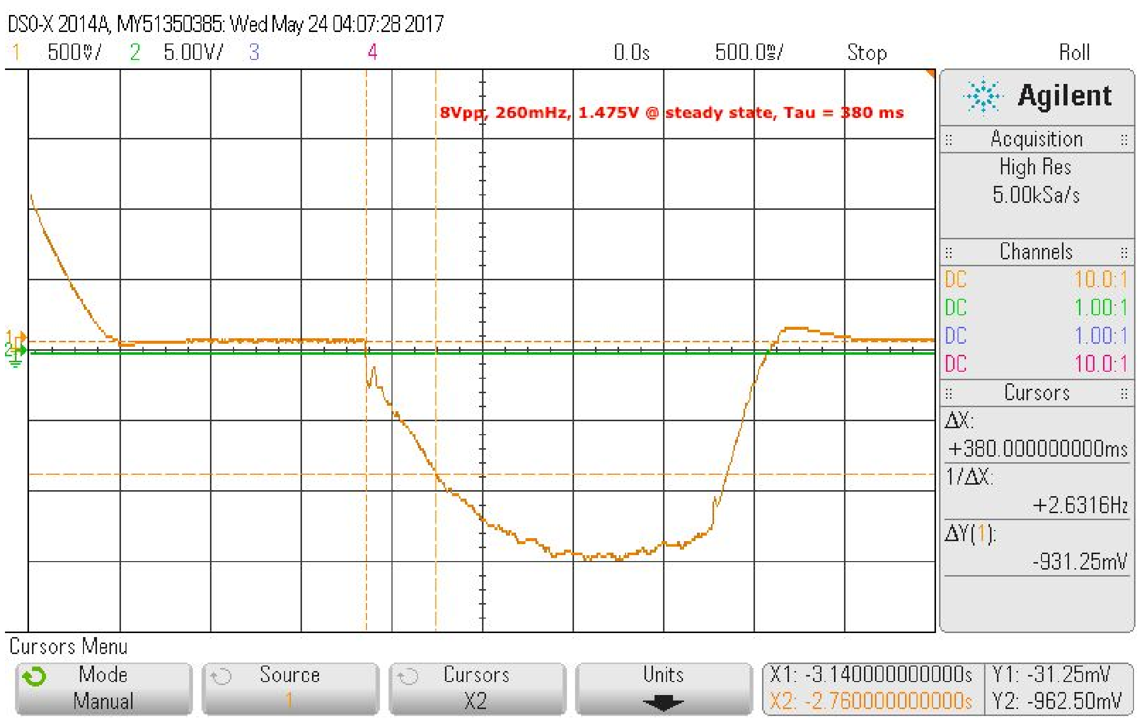

Figure 5: Oscilloscope measurement of tachometer output voltage corresponding to motor velocity in response to a -4 V step input. The steady state output is 1.475 V and the time constant is around 380 ms.

We ultimately decided to model our DC motor with a time constant of **Tau = 380 ms** since this was the average of the three best responses we obtained. Therefore our DC motor's Transfer Function is:

$$G(s) = \frac{k}{(s + \frac{1}{0.38})}$$

This is the transfer function of the motor outputting the position of the duck car as opposed to the velocity. A motor transfer function is usually represented as taking in as input a voltage and outputting a velocity. However, we needed the motor block to output a position because that is the input to the sensor so **an integrator (1/s) must be added**. Additionally, because the gain of the sensor is negative the **motor gain has been negated** in order to maintain system stability. This results in an overall motor transfer function of:

$$G(s) = \frac{-k}{s(s + \frac{1}{0.38})}$$

<u>Obtaining the Motor Gain</u>

The gain of the motor was found by adjusting the reference voltage for the system and measuring the speed of the car to find the steady state velocity in order to relate voltage input to velocity output of the motor.

Table 2: Measurements used to determine the motor gain

voltage (V)	time (s)	distance (m)	Velocity (m/s)
5	2.81	2	0.711743772
6	2.19	2	0.913242009
7	1.8	2	1.111111111
8	2.63	4	1.520912548
9	2.5	4	1.6
10	2.01	4	1.990049751

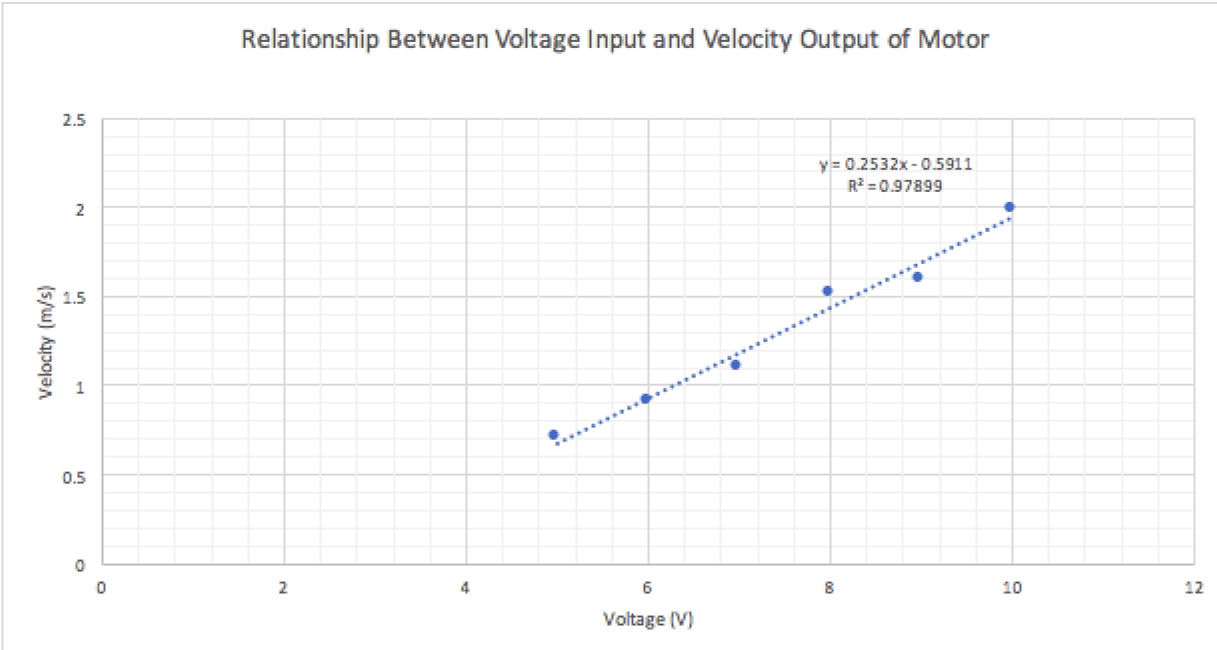

Figure 6: Velocity input to motor v. velocity of duck car

Figure 6 shows the linear relationship between voltage and velocity, using a line of best fit, we can see that the value of our motor gain is **k = 0.253**.

The Open-Loop Transfer Function

Using the block diagram in Figure 1, we can solve for our open-loop uncompensated Transfer Function:

$$G(s) * k_{sensor} = \frac{-0.25 * -3.48}{s\left(s + \frac{1}{0.38}\right)} = \frac{0.87}{s\left(s + \frac{1}{0.38}\right)}$$

III. Controller Design

Proportional Controller

We first started out with a proportional controller (P) to understand the response of our current closed-loop system and develop a baseline to improve upon. The settling time for this controller was very large and the system oscillated resulting in a system that did not stabilize unless it was physically stopped at the point at which error steady state is equal to zero. This distance was approximately 10 inches from the object at a reference voltage of 1.02 V.

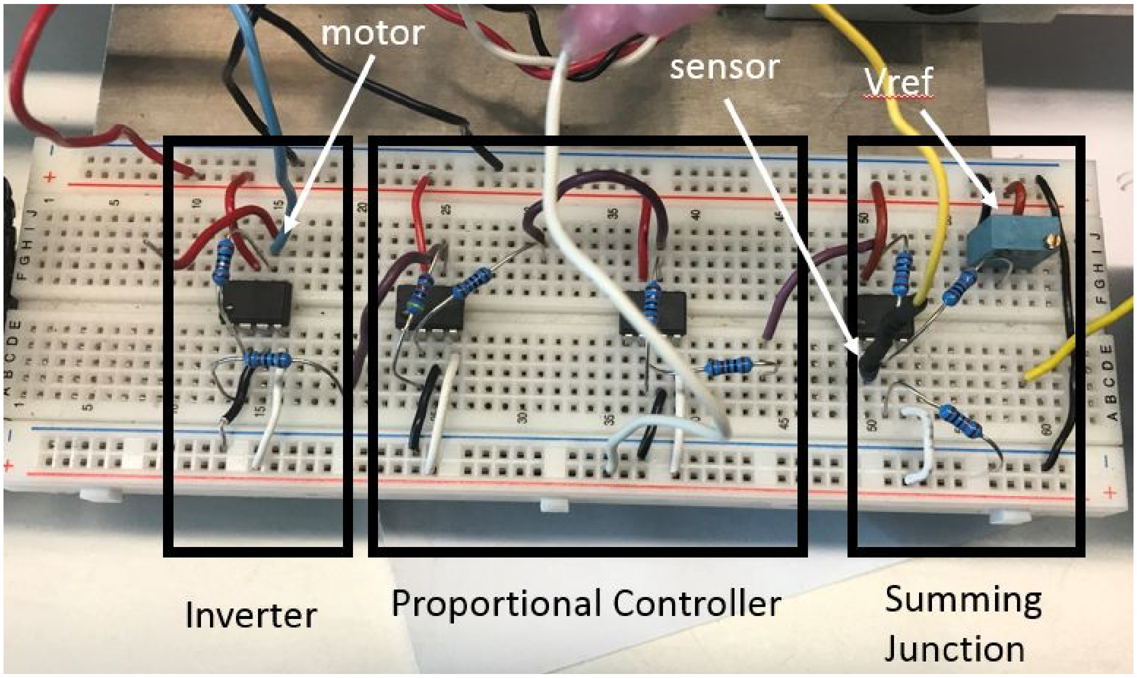

Figure 7: Circuit diagram for our proportional controller

In order to decrease the settling time, we knew we needed to have at least a zero. Therefore, using MATLAB, we modelled a few different types of compensators. We started out with a PID and settled on a Lead-Lag as our final compensator.

Other Controller Iterations

After designing a PD compensator with a real zero at s = -3 and a pole far from the origin we realized that we had to have a very high gain to get a system behavior close to the specifications that we wanted. This made the system saturated and therefore we were unable to use this PD compensator. Because of this, we considered using a PID or Lead-Lag compensator. After modelling several iterations of PID compensators on MATLAB's sisotool, we could not obtain a decent control effort for a system that met our specs with the PID compensator. Thus we settled on modelling a Lead-Lag compensator after building and testing our system with the PD and PID compensators.

Final Lead-Lag Controller

We ultimately found a lead-lag compensator using MATLAB's sisotool that would allow us to meet our settling time specification as well as our peak overshoot specification. The following table depicts the different lead-lag designs we implemented and tested.

Table 3: Resistor values for lead-lag controller

Design ID	Gain K = (R6*R4)/ (R5*R3)	R1 (ohms)	R2 (ohms)	R3 (ohms)	R4 (ohms)	R5 (ohms)	R6 (ohms)	C1 (uF)	C2 (uF)
1	3	5499.1	17000	14750	16000	100	268.5	10	10
2	24	5499.1	17000	14750	16000	100	2000	10	10
3	16	5499.1	17000	14750	16000	100	1475	10	10
4	18.23	5499.1	17000	14750	16000	100	1681	10	10
5	16.85	5499.1	17000	14750	16000	100	1553	10	10

Design 1 was our first model, due to its small gain value, we could not get the system to overcome stiction effects, thus the first thing we did was increase the gain to see if the lead-lag would work in the first place. When we adjusted the gain to 24 we found that our system had an overshoot so we experimented with lower gain values and settled on Design 5 since it was well-damped and experienced minimal settling time and overshoot.

The step response of the selected compensator design: (see MATLAB script in Appendix)

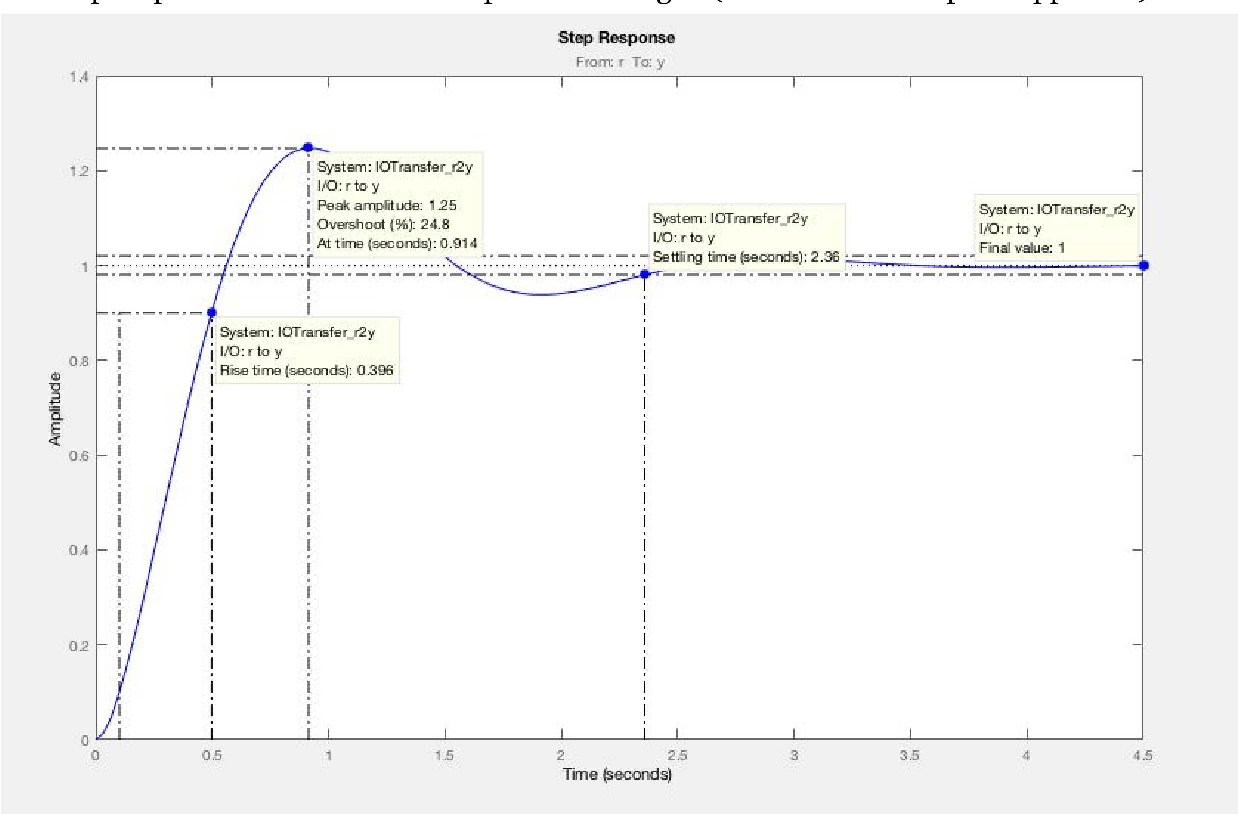

Figure 8: Step response of the compensated system which meets the specifications defined in section 1

The Bode Transfer Function:

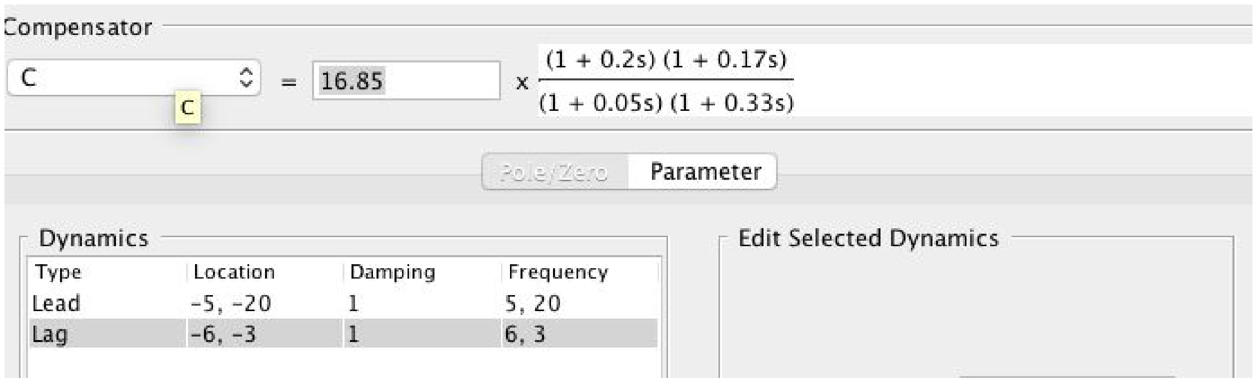

Figure 9: Transfer function of our lead-lag compensator generated in sisotool

This is the root locus plot of the compensated system, the specifications we defined in part 1 have been met (even though it's on the edge of meeting both the overshoot and settling time specifications).

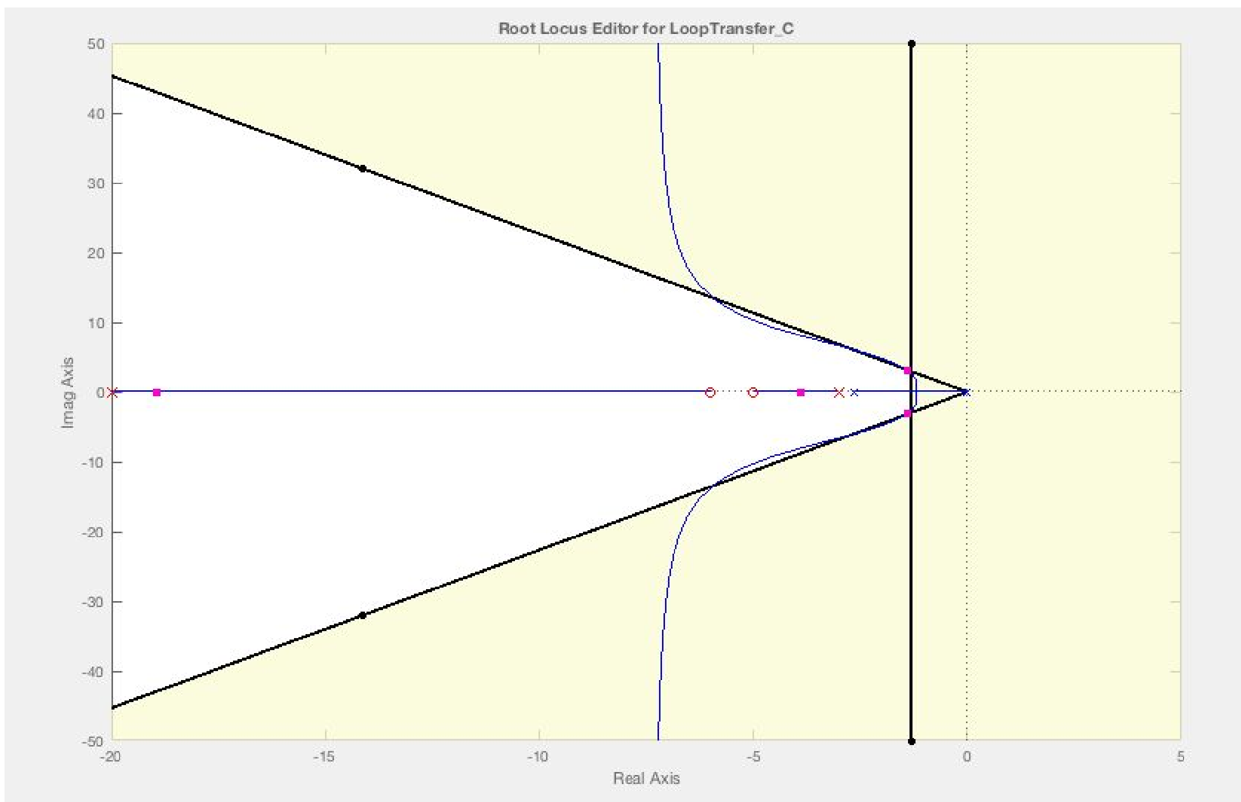

Figure 10: Root locus analysis used to find values for the lead-lag compensator and meet our specifications

The transfer function for the compensator is:

$$G_c(s) = 33.7\,\frac{(s+5)\,(s+6)}{(s+20)\,(s+3)}$$

Thus the overall open-loop compensated transfer function is:

$$G_{ol}(s) = 33.7\ \frac{29.31s^2 + 322.4s + 879.3}{s^4 + 25.63s^3 + 120.5s^2 + 157.9s}$$

And the overall closed-loop compensated transfer function is:

$$G_{cl}(s) = 33.7\ \frac{29.31s^2 + 322.4s + 879.3}{s^4 + 25.63s^3 + 149.8s^2 + 480.3s + 879.3}$$

Final block diagram, closed-loop

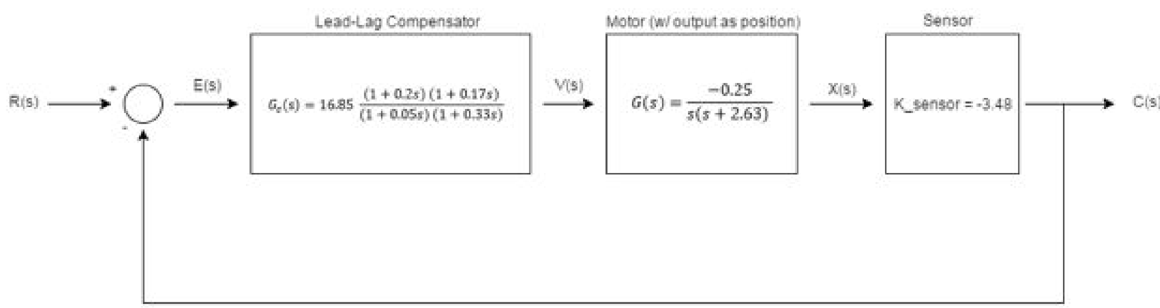

Figure 11: Final closed-loop transfer function

IV. Implementation of Compensator

<u>The Circuit Design</u>

Our circuit implementation is as follows with a Vref of 1.00 V corresponding to zero error steady state at 10.8 inches.

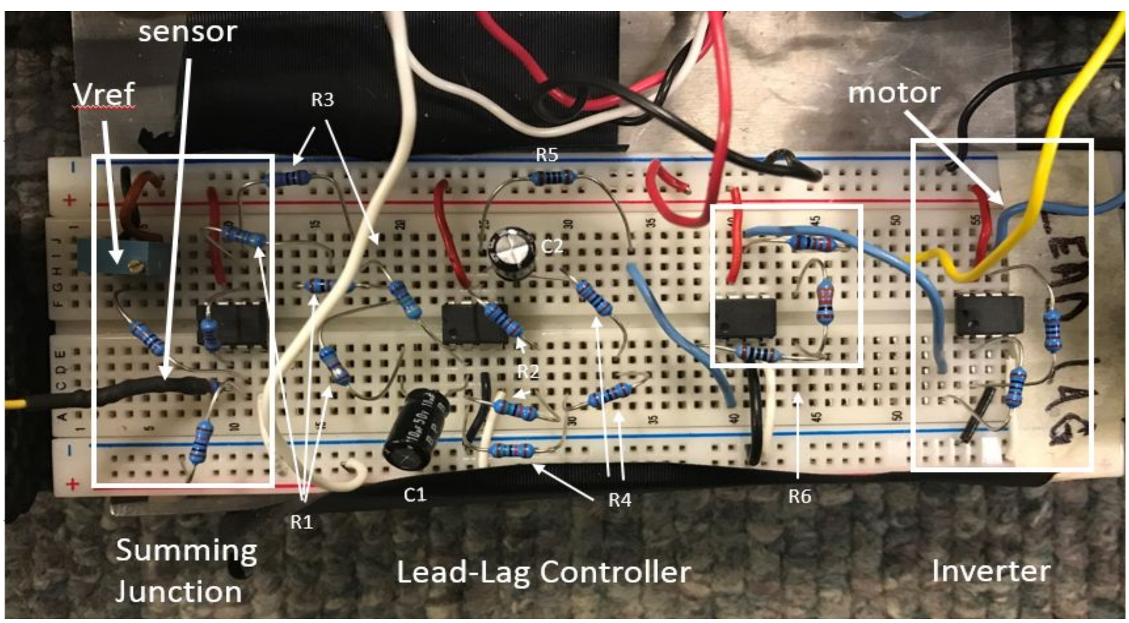

Figure 12: Final lead-lag compensator circuit

Summing Junction

Our summing junction had three resistors, each equal to 10K ohms, our reference voltage was fed through the non-inverting input of the summing junction op-amp and the output of the IR sensor goes into the inverting input of the summing junction op-amp.

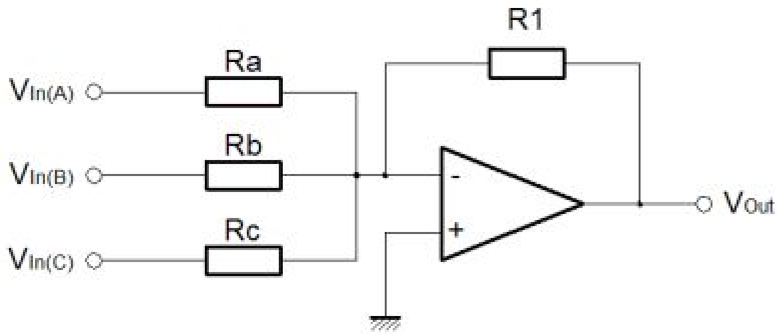

Figure 13: Summing junction circuit

Lead-Lag Controller

Our controller specified in Table 3 as Design 5 was implemented using two op-amps and capacitors at 10 uF, and resistors with the following values in ohms, R1 = 5499.1, R2 = 17000, R3 = 14750, R4 = 16000, R5 = 100, R6 = 1553.

$$\frac{R_6}{R_5}\frac{R_4}{R_3}\frac{[(R_1+R_3)C_1s+1](R_2C_2s+1)}{(R_1C_1s+1)[(R_2+R_4)C_2s+1]}$$

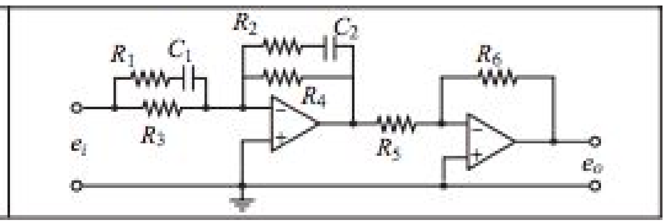

Figure 14: General lead-lag compensator circuit

Inverter

We had to add an inverter since the car, during initial testing we found that the car moved forward with positive voltage.

V. Discussion

First, we qualitatively analyzed the our compensated system. We could visually see that the system had minimal overshoot and a decent rise time when moving in reverse, particularly on the carpet where the tires had more traction. The rise time was significantly slower moving forward, but there was still minimal overshoot. We then wanted to experimentally verify that our system met all of the specifications that were analytically promised. We attempted to perform the same test we did on the uncompensated open-loop system to get an oscilloscope image for the step response of the motor on the compensated closed-loop system. However, it was difficult to get a good plot on the oscilloscope all the way up through steady state. The wires coming from the function generator and oscilloscope limited the car's movement and kept it from being able to reach steady state. This is visible in the plot below which shows the motor output for a 2V step input.

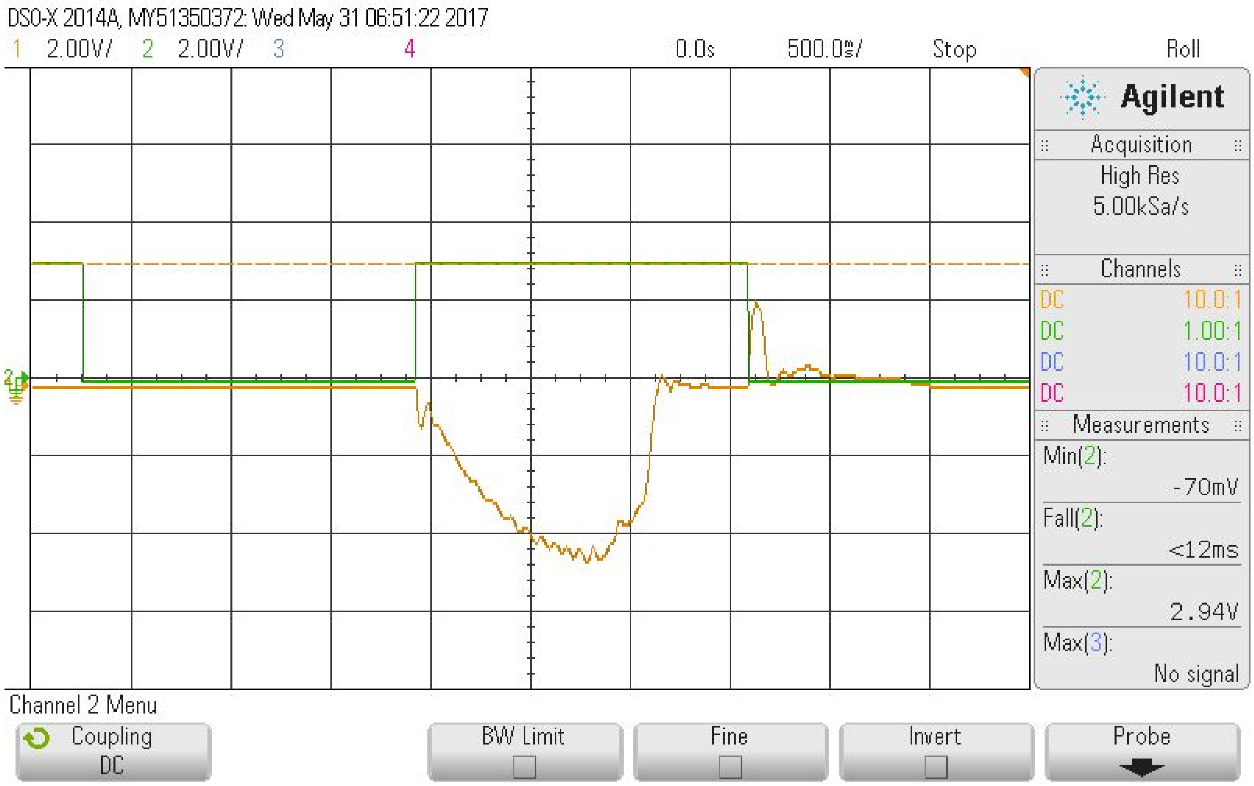

Figure 15: Oscilloscope image of attempt to attain the step response of the closed loop system

So next we measured the output of the sensor instead to gauge how position changed as a function of the input voltage shown in Figure 16. We attached the oscilloscope probe to the sensor, placed the duck car up against a white sheet so that we could first see the reverse response of the duck car, then ran the duck car at our standard reference voltage of 1.00 V. Immediately when the car settled at the distance corresponding to our reference voltage, we placed another obstacle 5 inches in front of the first sheet causing the duck car to retreat again. The oscilloscope plot below shows the output of the sensor from the test. It shows the effect of a disturbance or change in the location of an object and how quickly our system is able to respond to it. This took approximately 570 ms for our system to correct its position for this change in object location.

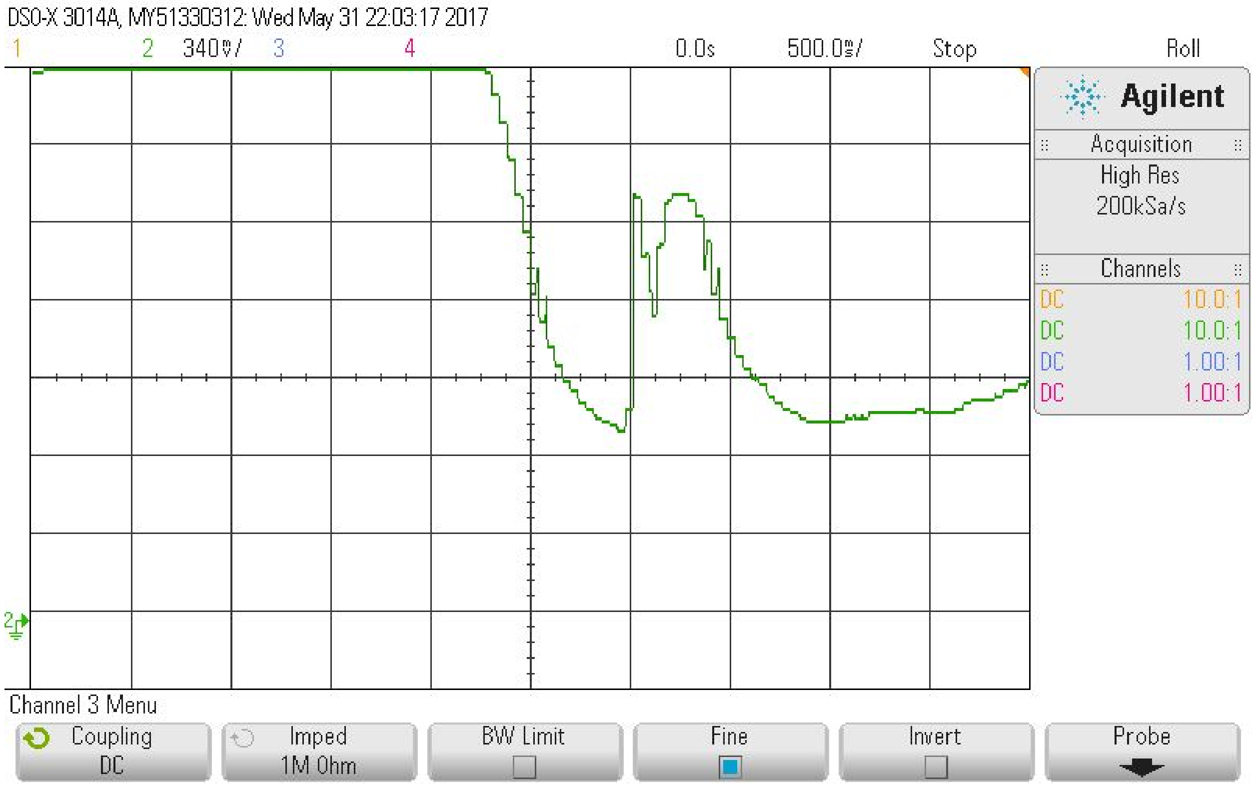

Figure 16: Oscilloscope image to gather response

Not only did this lead-lag compensator respond to objects placed in front of it, it also satisfied the settling time and peak overshoot specification we had defined in Section 1. Our compensated system has a settling time of 2.36 seconds and a peak overshoot of 24.3% as shown in Figure 7. It also has an error steady state equal to zero for a step input since there exists an integrator in the system. The bode plot generated in sisotool shows that the gain margin is infinity and that the phase margin is 44.1 degrees. Both of these values meet the minimum gain and phase margin specifications defined. We can also see that this system is stable in Figure 17. The closed-loop bandwidth is ~0.1 rad/sec which is as close to the maximum value it can obtain in this system.

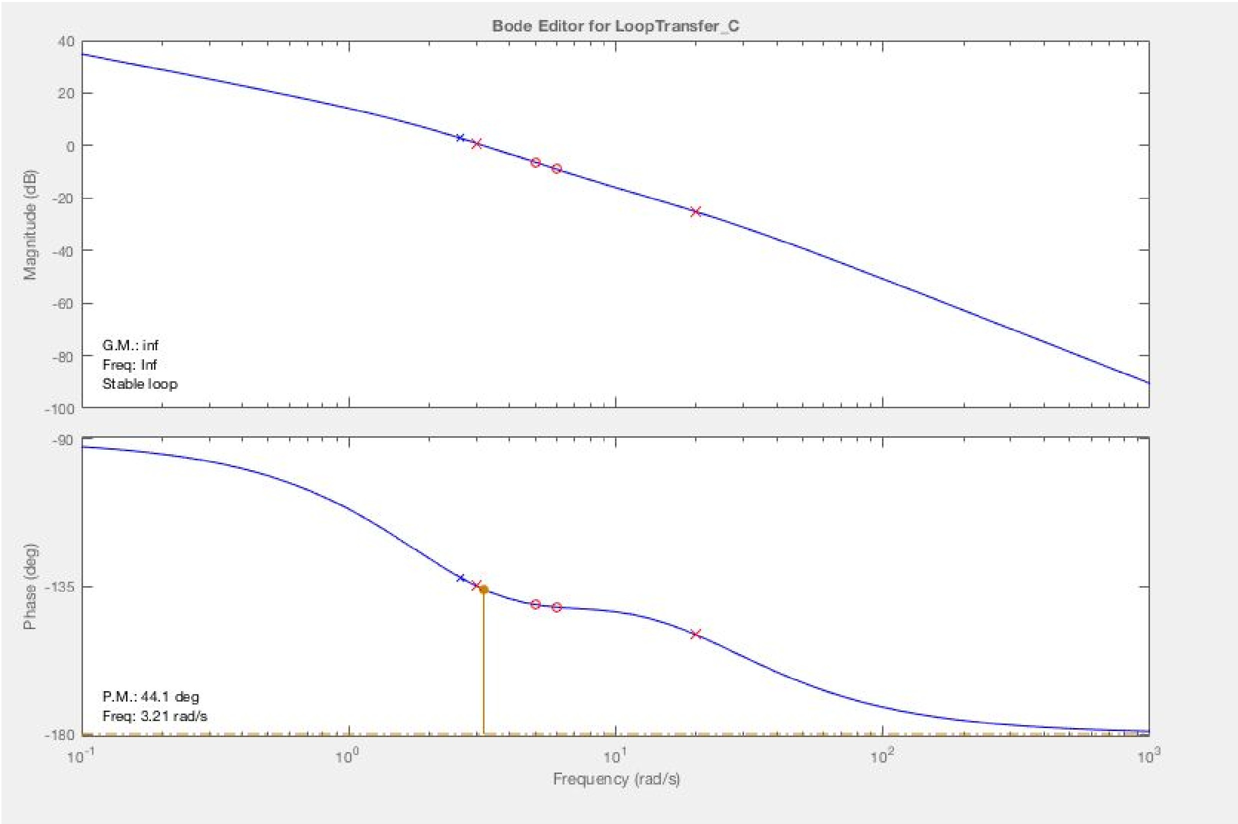

Figure 17: Bode plot of compensated system generated in sisotool

Our system is also able to respond to uncertainty and disturbances, we were able to visualize this when testing the compensated duck car and see how it responds to changes in the position of the reference object. It is able to respond decently quickly with very little overshoot based on the distance of the reference object from the IR sensor.

Along the process of designing multiple types of compensators, we quickly discovered that having a control effort that is not very large is important due to the limitation of the power amp on the car. Our current system has a max control effort of around 35 V.

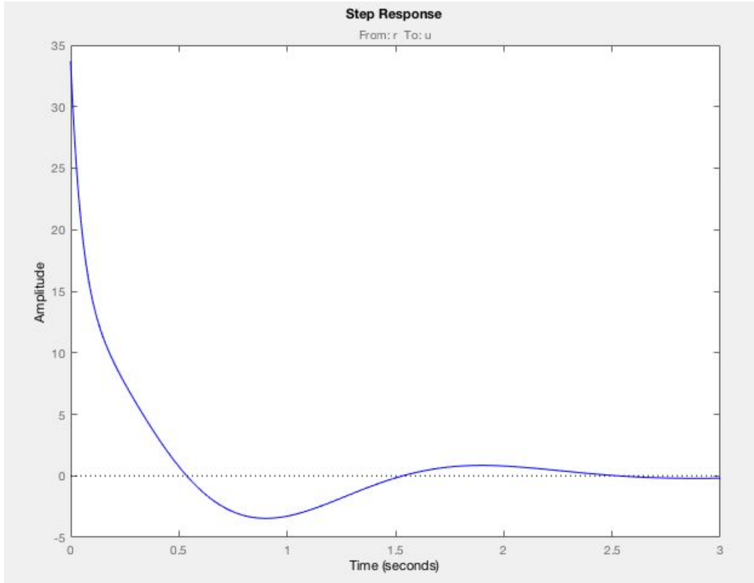

Figure 18: Control effort of compensated system generated in sisotool

Appendix

MATLAB script for finding compensator:

```
ksensor = 3.479;
ksys = 0.25;
Tau = 0.38;
num = ksys * ksensor;
denom = [1 1/Tau 0]
olsys= tf(num, denom);
%sisotool(olsys);

% add compensator transfer function
K = 33.7;
cnum = [1 11 30]
cden = [1 23 60]
colsys = tf(K *cnum, cden);

%full open loop tf
ol = olsys *colsys;

%full closed loop tf
cl = feedback(ol, 1);
```